SECONDARY MATHEMATICS III

IN SECONDARY MATHEMATICS III students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into four critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions. They expand their study of right triangle trigonometry to include general triangles. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

CRITICAL AREA 1: Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

CRITICAL AREA 2: This area develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

CRITICAL AREA 3: Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles opens up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

CRITICAL AREA 4: Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph

always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, adjust parameters to improve the model, and compare models by analyzing the appropriateness of the fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this area. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in Secondary Mathematics III describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

- **Standard SIII.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard SIII.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard SIII.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard SIII.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard SIII.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard SIII.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

- **Standard SIII.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see* $5 - 3(x - y)^2$ *as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

- **Standard SIII.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—The Complex Number System (N.CN)

Use complex numbers in polynomial identities and equations. Build on work with quadratic equations in Secondary Mathematics II **(Standards N.CN.8–9).**

- **Standard N.CN.8** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.

- **Standard N.CN.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Limit to polynomials with real coefficients.

Strand: ALGEBRA—Seeing Structures in Expressions (A.SSE)

Interpret the structure of expressions. Extend to polynomial and rational expressions **(Standards A.SSE.1–2).** Write expressions in equivalent forms to solve problems **(Standard A.SSE.4).**

- **Standard A.SSE.1** Interpret polynomial and rational expressions that represent a quantity in terms of its context.★
  - **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
  - **b.** Interpret complex expressions by viewing one or more of their parts as a single entity. *For example, examine the behavior of* $P(1+r/n)^{nt}$ *as* n *becomes large.*

- **Standard A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

- **Standard A.SSE.4** Understand the formula for the sum of a series and use the formula to solve problems.
  - **a.** Derive the formula for the sum of an arithmetic series.
  - **b.** Derive the formula for the sum of a geometric series, and use the formula to solve problems. Extend to infinite geometric series. *For example, calculate mortgage payments.*★

Strand: ALGEBRA—Arithmetic With Polynomials and Rational Expressions (A.APR)

Perform arithmetic operations on polynomials, extending beyond the quadratic polynomials **(Standard A.APR.1).** Understand the relationship between zeros and factors of polynomials **(Standards A.APR.2–3).** Use polynomial identities to solve problems **(Standards A.APR.4–5).** Rewrite rational expressions **(Standards A.APR.6–7).**

- **Standard A.APR.1** Understand that all polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- **Standard A.APR.2** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
- **Standard A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **Standard A.APR.4** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
- **Standard A.APR.5** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers. *For example, with coefficients determined by Pascal's Triangle.*
- **Standard A.APR.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division or, for the more complicated examples, a computer algebra system.
- **Standard A.APR.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Strand: ALGEBRA: CREATING EQUATIONS (A.CED)

Create equations that describe numbers or relationships, using all available types of functions to create such equations **(Standards A.CED.1–4).**

- **Standard A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- **Standard A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **Standard A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in

a modeling context. *For example, maximizing the volume of a box for a given surface area while drawing attention to the practical domain.*

- **Standard A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange the compound interest formula to solve for* t: $A = P(1+r/n)^{nt}$

Strand: ALGEBRA: REASONING WITH EQUATIONS AND INEQUALITIES (A.REI)

Understand solving equations as a process of reasoning and explain the reasoning **(Standard A.REI.2).** Represent and solve equations and inequalities graphically **(Standard A.REI.11).**

- **Standard A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

- **Standard A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ *and* $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, for example, using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Interpret functions that arise in applications in terms of a context **(Standards F.IF.4–6).** Analyze functions using different representations **(Standards F.IF.7–9).**

- **Standard F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

- **Standard F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble* n *engines in a factory, then the positive integers would be an appropriate domain for the function.*★

- **Standard F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

- **Standard F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

  b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Compare and contrast square root, cubed root, and step functions with all other functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.

■ **Standard F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

■ **Standard F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Strand: FUNCTIONS—Building Functions (F.BF)

Build a function that models a relationship between two quantities. Develop models for more complex or sophisticated situations **(Standards F.BF.1).** Build new functions from existing functions **(Standards F.BF.3–4).**

■ **Standard F.BF.1** Write a function that describes a relationship between two quantities.★

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

■ **Standard F.BF.3** Identify the effect on the graph of replacing *f(x) by f(x) + k, k f(x), f(kx), and f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Note the effect of multiple transformations on a single function and the common effect of each transformation across function types. Include functions defined only by a graph. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

■ **Standard F.BF.4** Find inverse functions.

a. Solve an equation of the form *f(x) = c* for a simple function *f* that has an inverse and write an expression for the inverse. Include linear, quadratic, exponential, logarithmic, rational, square root, and cube root functions. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.

Strand: FUNCTIONS—Linear, Quadratic, and Exponential Models (F.LE)

Construct and compare linear, quadratic, and exponential models and solve problems **(Standards F.LE.3–4).** Interpret expressions for functions in terms of the situation it models.

Introduce $f(x) = e^x$ as a model for continuous growth **(Standard F.LE.5).**

- **Standard F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quanitity increasing linearly, quadratically, or (more generally) as a polynomial function.
- **Standard F.LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. Include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log $xy = log\ x + log\ y$.
- **Standard F.LE.5** Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Extend the domain of trigonometric functions using the unit circle **(Standards F.TF.1–3).** Model periodic phenomena with trigonometric functions **(Standards F.TF.5–7).**

- **Standard F.TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **Standard F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- **Standard F.TF.3** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for *x,* where *x* is any real number.
- **Standard F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
- **Standard F.TF.7** Use inverse functions to solve trignometric equations that arise in modeling context; evaluate the solutions using technology and interpret them in terms of context. Limit solutions to a given interval. ★

Strand: GEOMETRY—Similarity, Right Triangles, and Trigonometry (G.SRT)

Apply trigonometry to general triangles. With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles **(Standards G.SRT.9–11).**

- **Standard G.SRT.9** Derive the formula $A = ½\ ab\ sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- **Standard G.SRT.10** Prove the Laws of Sines and Cosines and use them to solve problems.

- **Standard G.SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Visualize relationships between two-dimensional and three-dimensional objects **(Standards G.MD.4).**

- **Standard G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Strand: GEOMETRY—Modeling With Geometry (G.MG)

Apply geometric concepts in modeling situations **(Standards G.MG.1–3).**

- **Standard G.MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
- **Standard G.MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
- **Standard G.MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Strand: STATISTICS—Interpreting Categorical and Quantitative Data (S.ID)

Summarize, represent, and interpret data on a single count or measurement variable. While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution **(Standard S.ID.4).**

- **Standard S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Strand: STATISTICS—Making Inferences and Justifying Conclusions (S.IC)

Understand and evaluate random processes underlying statistical experiments **(Standard S.IC.1).** Draw and justify conclusions from sample surveys, experiments, and observational studies. In earlier grades, students are introduced to different ways of collecting data and use

graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. For S.IC.4, focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness **(Standards S.IC.3–4, 6).**

- **Standard S.IC.1** Understand that statistics allow inferences to be made about population parameters based on a random sample from that population.
- **Standard S.IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
- **Standard S.IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- **Standard S.IC.6** Evaluate reports based on data.