

## Solving Inequalities

| <b>Interval Notation</b>   |                          |              |
|--|--------------------------|--------------|
| The symbols [ ] mean “included” or “closed” while symbols ( ) mean “not included” or “open.” |                          |              |
| <b>Inequality</b>  | <b>Interval Notation</b> | <b>Graph</b> |
| $-2 < x < 5$   | $(-2, 5)$                |              |
| $1 \leq x \leq 7$  | $[1, 7]$                 |              |
| $-4 \leq x < 4$  | $[-4, 4)$                |              |
| $0 < x \leq 3$   | $(0, 3]$                 |              |
| $x < -3$   | $(-\infty, -3)$          |              |
| $x \geq 1$   | $[1, \infty)$            |              |

**Note:** When multiplying or dividing by a negative number remember to reverse the inequality sign.

**Example 1:** Solve  $x + 7 > 9$  and graph the solution on a number line. Write the answer in interval notation.

$$x + 7 > 9$$

$$x + 7 - 7 > 9 - 7$$

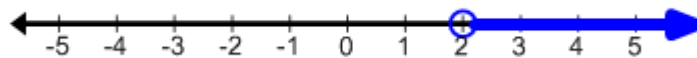
Subtract 7 from each side of the inequality to isolate the variable.

$$x > 2$$

Simplify.

$$(2, \infty)$$

Interval notation.



**Example 2:** Solve  $\frac{d}{3} - 8 \leq -10$  and graph the solution on a number line. Write the answer in interval notation.

$$\frac{d}{3} - 8 \leq -10$$

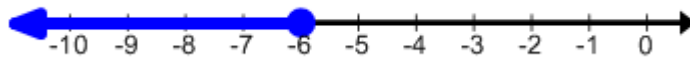
$$\frac{d}{3} - 8 + 8 \leq -10 + 8 \quad \text{Add 8 to each side of the inequality.}$$

$$\frac{d}{3} \leq -2 \quad \text{Simplify.}$$

$$3 \cdot \frac{d}{3} \leq -2 \cdot 3 \quad \text{Multiply each side of the inequality by 3.}$$

$$d \leq -6 \quad \text{Simplify.}$$

$$(-\infty, -6] \quad \text{Interval notation.}$$



**Example 3:** Solve  $-4x + 3 < 19$  and graph the solution on a number line. Write the answer in interval notation.

$$-4x + 3 < 19$$

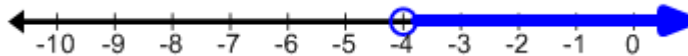
$$-4x + 3 - 3 < 19 - 3 \quad \text{Subtract 3 from each side of the inequality.}$$

$$-4x < 16 \quad \text{Simplify.}$$

$$\frac{-4x}{-4} < \frac{16}{-4} \quad \text{Divide each side of the inequality by -4.}$$

$$x > -4 \quad \text{Reverse the inequality, and simplify.}$$

$$(-4, \infty) \quad \text{Interval notation.}$$



## Solving Multi-step Inequalities

**Example 4:** Solve  $4(5x+3)-5 > -13$  write the answer in interval notation.

$$4(5x+3)-5 > -13$$

$$20x+12-5 > -13$$

Distribute the 4 to both  $5x$  and  $3$ .

$$20x+7 > -13$$

Simplify each side of the inequality.

$$20x+7-7 > -13-7$$

Subtract 7 from each to each side of the inequality.

$$20x > -20$$

Simplify.

$$\frac{20x}{20} > \frac{-20}{20}$$

Divide each side of the inequality by 20.

$$x > -1$$

Simplify.

$$(-1, \infty)$$

Interval notation.

**Example 5:** Solve  $5+4x \leq x+8$  write the answer in interval notation.

$$5+4x \leq x+8$$

$$5+4x-x \leq x-x+8$$

Subtract  $x$  from each side of the inequality.

$$5+3x \leq 8$$

Simplify.

$$5-5+3x \leq 8-5$$

Subtract 5 from each side of the inequality.

$$3x \leq 3$$

Simplify.

$$\frac{3x}{3} \leq \frac{3}{3}$$

Divide each side of the inequality by 3.

$$x \leq 1$$

Simplify.

$$(-\infty, 1]$$

Interval notation.

## Solving Compound Inequalities

**Example 6:** Solve the compound inequality  $4x - 3 > 9$  or  $-2x > 2$ . Write the answer in interval notation.

$$4x - 3 > 9 \text{ or } -2x > 2$$

$$4x - 3 > 9$$

$$4x > 12 \quad \text{or} \quad -2x > 2$$

$$x > 3 \quad \quad \quad x < -1$$

$$(-\infty, -1) \cup (3, \infty)$$

Solve each inequality separately.

This is an “or” compound inequality so it is a union of the solutions.

**Example 7:** Solve the compound inequality  $-4 < x + 1 < 6$ . Write the answer in interval notation.

$$-4 < x + 1 < 6$$

$$-4 < x + 1$$

$$x + 1 < 6$$

$$-4 - 1 < x + 1 - 1 \quad \text{and} \quad x + 1 - 1 < 6 - 1$$

$$-5 < x$$

$$x < 5$$

$$-5 < x < 5$$

$$(-5, 5)$$

Write the compound inequality as two inequalities. Solve each inequality separately.

Put the compound inequality back together.

This is an “and” compound inequality so it is the intersection of the two solutions.